

# An agent model of conversation

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**Abstract**—The aim of this paper is to develop a model of the temporal (not contentual) evolution of conversation between two agents. Our target variable is the waiting time between utterances. We develop a computational version of the turn taking model by Sacks et. al. [6] which was seminal to the formation of conversation analysis in sociology. Our interest is in understanding the mechanism by which agents cooperatively control the flow and intensity of conversation. We find that our model is both realistic as well as amenable to mathematical analysis. Moreover, it facilitates the automated analysis of large sets of conversational data that can be found e.g. in the social media.

## I. INTRODUCTION

Conversations are an important object of study to anyone interested in understanding social and cooperative behavior. This is because conversation requires a number of adaptive procedures to be performed by the participants which are directed towards common understanding about the way a topic should be approached. Two minds have to collaborate to achieve discourse coherence and “make progress” on the subject of their discussion. It is in fact generally agreed among social scientists that conversation is fundamentally a cooperative endeavor and that the analysis of conversations provides important insights into the basic mechanisms of human intellectual interaction.

These interactive mechanisms include nonverbal communication (see e.g. [1]), and the idea of an interaction *order* introduced by Goffmann [2] which has been pursued by the research program of conversation analysis (CA). CA studies the way interaction order is established in practice, in particular how people take turns at talk, how they deal with overlaps and interruptions and how the sequence of utterances (and more general [speech] actions) is organized. CA argues that the “...meaning of an action is heavily shaped by the sequence of actions from which it emerges, and that the social context is dynamically created [...] through the sequential organization of interaction”, see [3], p.223. From a cognitive perspective, any statement has to signal understanding of the preceding statements and prepare the floor for the next in order to establish coherence. This means that

“each sentence [...] must contain some direct or indirect indication as to how it fits into the stream of talk”, see [4], p.119.

The aim of this paper is to formalize some of the propositions of CA and to propose a mathematical model of sequential interaction which could serve as a building block in an automated (machine learning) framework of discourse analysis. In terms of concepts, we build on the work by Schegloff [5] while following the seminal ideas introduced by Sacks et. al. in [6]. We will allow for deviations from the established practices as we are ultimately interested in extracting machine-“readable” features out of a large body of conversation data. In particular, we will not only be focused on spoken conversation but broaden the scope to social media content, as retrieved in blogs and instant messaging services which display a similar level of immediacy of turns at talk than classical (personal) conversation, see e.g. [7].

## II. TURN CONSTRUCTIONAL UNIT, ADJACENCY PAIRS AND THE PROBLEM OF DETECTING CONVERSATION

An important viewpoint assumed by CA is that conversation may be understood more from what it is *doing* than from what it is *about*. In other words, turns at talk are viewed as implementing specific courses of action rather than being a mere conveyor of information. As an example (see [5]), the expression “Would you like some tea?” is better understood as “doing an offer” than as “about tea”. This idea is formalized as follows: turns-at-talk are composed of so-called turn constructional units (TCUs), the fundamental building blocks of conversation. The definition and scope of TCUs is that they realize actions, for example an offer, a greeting, a request, a complaint etc. Upon completion of the action, the conversation reaches a transition relevance place (TRP) where the turn may pass to another speaker. It may, however, happen that a speaker talks past this place by starting another TCU or by extending the first TCU. At some point, the turn is allocated to the next speaker who either self-selects this role or is being appointed by the first speaker. Once a turn has been successfully passed on to another

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speaker an *adjacency pair* is formed. Adjacency pairs constitute the central organizing format of the sequences of interaction in which we are interested. They consist of two turns by two different speakers which are relatively ordered. The so-called “first pair part” initiates the exchange (i.e. it performs an *action* as described above) whereas the “second pair part” *responds* to this action by providing a relevant follow-up statement. In this paper, we assume that the responses are always “pair-type related”, i.e. we will not allow improper pairings such as “Would you like some tea?”–“Hi!”. We then propose to study conversations by looking at the time pattern of speech occurrences of the two parties. We postulate that the “liveliness” of the conversation may be determined by looking at occurrence times only, i.e. without explicit reference to the content. In particular, we put forward the following

*Problem statement:* Given an unlabeled set of utterances, determine whether or not the corresponding speakers are involved in a conversation.

By “unlabeled” we mean that neither the identity of speakers nor the exact content is specified. The motivation behind this assumption is that we aim at understanding semi-anonymous electronic conversations occurring in the blogosphere, i.e. the space of all web-blogs. In this space, conversations start spontaneously, e.g. with the statement of an opinion, and then gradually evolve into topics as new participants enter the discussion. In other words, topics are not given but an emergent property. It may thus prove difficult to understand social media conversations based on topical cues only. The timing of messages is a critical complementary information. Our hypothesis is that, if indeed agents cooperate in order to establish coherence, these efforts should be reflected not only in the content but also in the time dimension of the data.

In a personal conversation, the response times are well-defined and any failure to provide an adequate and timely reaction will result in a breakdown of the conversational flow. In electronic “conversation” the time grid is somewhat looser and has, in fact, a random component. But if an e-mail remains unanswered for a long interval of time the conversational flow will eventually fade. From daily experience one might even be tempted to say that it is better to respond promptly and less accurately than the opposite way. On the other hand, an excessive activity on the side of one participant may deter others from contributing. In other words, there is a *right* timing of utterances – and agents have to adapt to each other in

order to find it. This brings us to the specific objective of this paper.

*Objective:* Develop a generative model of the waiting times between utterances in a conversation.

The idea is to use this model in order to provide insights to the more general problem stated above: the waiting times obtained in the context of conversation are different from those obtained in multiple but unrelated streams of talk. In other words, waiting times in a conversation reflect the effort of every participant to create, maintain or renew the flow of ideas.

### III. THE CONVERSATION MODEL

We will think of the blogosphere (the source of our data) as an e-mail sent *from* and *to* multiple recipients. This means that we have to distinguish two types of agents – sender and respondent – who may, however, interchange their role as the conversation develops. Agents produce events (an email sent) at discrete instants of time  $t \in \mathbb{N}_0^+$ . This means that event times form a subsequence  $\{t_k\}_{k \geq 0}$  where  $k$  refers to an event produced by either agent one or two.

Let  $\{t_{k_1}\}$  and  $\{t_{k_2}\}$  be the event times belonging to agent one and two respectively, so that  $\{t_k\} = \{t_{k_1}\} \cup \{t_{k_2}\}$ . We assume that the occurrence of events is governed by a stochastic process: We count events as we count successes in a number of Bernoulli trials and obtain an expression for the waiting time until the first success as follows:

$$P(W_i) = q_i (1 - q_i)^{W_i - 1} \quad i = 1, 2 \quad (1)$$

where  $q_i$  is the probability of success for one Bernoulli trial performed by agent  $i$ . In our case, the trials are repeated at every (discrete) instant of time. The key ingredient in our model is that the probabilities are time-varying quantities. This time-variation is in the spirit of a (mutually exciting) Hawkes process [8] and serves to implement the structural elements of successful turn-taking described above. In particular, we implement the *adjacency pair* as the central device for the organization of sequences. In our model, this means that

- TCUs are represented as stochastic events (corresponding to the sending of a message).
- Agents produce events with (success) probability  $q_i$  which may assume two values: “high” and “low”.
- Agents adhere to a strict turn-taking schedule: after agent 1 produces an event, the event probability  $q_1$  switches to its “low” level until agent 2 produces

at least one event. Once this occurs,  $q_1$  is set back to its “high” level. The same ruling applies to  $q_2$  of agent 2.

If the high probability level is close to one and the low level close to zero the schedule creates a regular, periodic exchange where every question (the first pair part) is being answered (second pair part) which, in turn, gives rise to a new question. Our model is narrow in the sense that it reproduces relatively formal, dyadic encounters in which conversation proceeds in this orderly, “one-at-a-time” fashion. See [9] for a critique of the Sacks–Schegloff–Jefferson model which would also apply in our case. Deviations from the regular pattern occur if the two probability levels are both closer to 0.5. In this case, agent 1 may produce multiple events (one generated from a stochastic process using the high success probability, and the others generated using the low success probability). In any case,  $q_1$  will remain in the “low” state until agent 2 has had a chance to express him- or herself. At this point,  $t_{k_2}$  is the last event time in our conversation “system” and we have that  $t_{k_2} > t_{k_1}$ . With this understanding we write

$$q_1(t_{k+1}) = \bar{q}_1 + a_1 1_{[t_{k_2} > t_{k_1}]} \quad (2)$$

$$q_2(t_{k+1}) = \bar{q}_2 + a_2 1_{[t_{k_1} > t_{k_2}]} \quad (3)$$

where  $\bar{q}_i > 0$  represents the low (base) probability and  $a_i$  the “step-up” level which satisfies  $0 \leq a_i \leq (1 - \bar{q}_i)$  so that  $0 < q_i(t_{k+1}) \leq 1$ ,  $i = 1, 2$ . The Boolean expression  $1_{[\cdot]}$  switches to one once the condition in brackets is met.

Resuming our above example, equation (2) refers to the situation where agent 1 has sent a message and agent 2 either has not yet responded (in which case  $q_1$  remains at the low level  $\bar{q}_1$ ) or agent 2 has responded (in which case  $t_{k_2} > t_{k_1}$  and  $q_1$  is set to the higher level  $\bar{q}_1 + a_1$ ). It is important to notice that the equation determines the probability of the *next* potential event. If  $t > 0$  denotes current (natural) time the next event occurs at  $t_{k+1} > t$  while the sending of a message by agent 1 (which has led to the situation) occurred at  $t_k \leq t$ . In other words, the equation establishes a relation between  $t_k$  and  $t_{k+1}$ . Loosely speaking, the system retains memory of the last event produced by agent 1 (which is nonlinear function of  $q_1(t_k)$  and  $q_2(t_k)$  as we shall see below). We may therefore think of equation (2) as a state equation which describes a single transition from the low to the high level of  $q_1$  (and back). The transition takes place at the first occurrence  $k_2^*$  of an agent 2–event. The key here is that this event occurs *following* an agent 1–event and may therefore be associated with a “response” action.

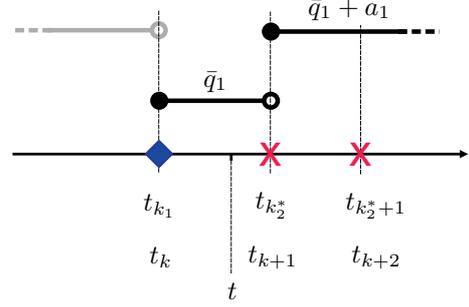


Fig. 1. Illustration of the sequence of events encoded in equation (2)

In order to detect it we need to retain memory of the fact that agent 1 has sent a message. This represents the very limited “eigendynamics” contained in equation (2). Further responses sent by agent 2 at instants  $t_{k_2^* + \kappa} > t_{k_1}$ ,  $\kappa = 1, 2, \dots$  do not change the relation in the Boolean expression and thus have no impact on  $q_1(t_{k+1})$  until, again, agent 1 speaks. The equivalent relationship hold for agent 2. Concurrent events  $t_{k_1} = t_{k_2}$  are excluded from our model as we shall see below.

In a second step, we endow our agents with the ability to “listen”: after sending a message, the agents count the instants of time until an “answer” is received. Relating again to equation (2), there will be a block of zeros in the Boolean operator  $1_{[t_{k_2} > t_{k_1}]}$  followed by a block of ones (after time  $t_{k_2^*}$ ) which completes an adjacency pair according to our schedule. The number of zeros in the first block corresponds to the waiting time for a response. We claim that this counting is done unconsciously and used to coordinate the timing of messages. It provides agent 1 with an estimate of the event probabilities employed by agent 2. Indeed, let

$$\hat{q}_2(t_{k_2^*}) = \left[ \frac{1}{n_2^*} \sum_{\substack{\bar{t}_{k_1} < \bar{t}_{k_2^*} \\ \bar{t}_{k_2^*} \leq t_{k_2^*}}} \bar{t}_{k_2^*} - \bar{t}_{k_1} \right]^{-1} \quad (4)$$

be the inverse of the average response times. We refer to figure (1) in order to visualize one such interval  $[\bar{t}_{k_1}, \bar{t}_{k_2^*}]$  and denote by  $n_2^*$  the number of intervals observed up to the point  $t_{k_2^*}$ .  $\hat{q}_2(t_{k_2^*})$  is agent 1’s estimate of  $\bar{q}_2 + a_2$ , i.e. the “high” level of  $q_2$ . The estimate can be shown to maximize the likelihood function.

*Lemma:* Let  $a_2$  and  $\bar{q}_2$  in (3) be constants.  $\hat{q}_2(t_{k_2^*})$  as defined in (4) is agent 1’s maximum likelihood estimate of  $\bar{q}_2 + a_2$ .

*Proof:* Let  $R_2 = \bar{t}_{k_2^*} - \bar{t}_{k_1}$  be a (stochastic) response time. Its distribution is given by (1) using  $q = q_2 = \text{const.}$  (corresponding to the “high” probability level). Let  $\bar{r}_2 = \{r_{21}, r_{22}, \dots, r_{2n_2^*}\}$  be a set of realizations. The likelihood function is given by

$$\mathcal{L}(q_2 | \bar{r}_2) = q_2^{n_2^*} \prod_{i=1}^{n_2^*} (1 - q_2)^{(r_{2i} - 1)} \quad (5)$$

The function is maximized at

$$q_2^* = \left[ \frac{1}{n_2^*} \sum_{i=1}^{n_2^*} r_{2i} \right]^{-1} \quad (6)$$

□

Note that an analogous result holds for agent 2’s best estimate  $\hat{q}_1(t_{k_1^*})$  of  $q_1$ .

In the third step, we specify when events take place: this occurs when natural time  $t$  equals event time  $t_k$ ,  $k \geq 0$ . The event processes of the two agents are coupled. In fact, the time–variation of the critical parameter  $q$  is driven by the stochastic occurrence of “partner” events. In order to simulate the process we first determine the time of the next event within the *system* and then decide to whom this event should be attributed. This procedure is an example of a stochastic simulation algorithm first introduced in the context of chemical reactions, see [10]. See also [11] for a recent example in bioinformatics. Let

$$t_k = t_{k-1} + w_k \quad (7)$$

be the time of an event in *either* message stream expressed as the time of a previous event  $t_{k-1}$  plus a realization  $w_k$  of the (global) waiting time. Let  $u_i(t_k) = \{0, 1\}$  indicate whether an event is produced by agent  $i$  at time  $t_k$ . The probability of seeing an event after waiting  $w_k$  instants of time is given by

$$P(\{u_1(t_k) = 1\} \cup \{u_2(t_k) = 1\}, W = w_k) = [1 - (1 - q_1)(1 - q_2)] \cdot [(1 - q_1)(1 - q_2)]^{w_k - 1} \quad (8)$$

provided that  $q_1$  and  $q_2$  is constant over the  $w_k$  instants. This is indeed the case as the probabilities are updated only at event times: as we have seen above, the Boolean function in equations (2) and (3) links the time–variation of  $q_1$  and  $q_2$  to events. In other words,  $q_1(t) = \text{const.}$  for  $t \in [t_{k-1}, t_k)$  and we denote this probability by  $q_1(t_k)$ . The same holds for  $q_2(t_k)$ . We obtain the cumulative distribution function:

$$P(\{u_1(t_k) = 1\} \cup \{u_2(t_k) = 1\}, W \leq w_k) = 1 - [(1 - q_1(t_k))(1 - q_2(t_k))]^{w_k} \quad (9)$$

This means that we may generate sample waiting times by drawing samples  $\nu_k$  from the uniform distribution  $\mathcal{U}[0, 1]$  and determine the *smallest* integer  $w_k$  that satisfies:

$$\nu_k \leq 1 - [(1 - q_1(t_k))(1 - q_2(t_k))]^{w_k} \quad (10)$$

Together with equation (7), we thereby obtain the time of the *next* event  $t_k$ . At  $t = t_k$  we draw a second sample  $\xi_k$  from  $\mathcal{U}[0, 1]$  and decide whether the event should carry the label 1 or 2:

$$t_k = \begin{cases} t_{k_1} & \text{if } \xi_k \leq \frac{q_1(t_k)}{q_1(t_k) + q_2(t_k)} \\ t_{k_2} & \text{otherwise} \end{cases} \quad (11)$$

which means that we have specified one of the sets in  $\{t_{k_1}\} \cup \{t_{k_2}\}$ . Note also that  $u_1(t_{k_1}) = 1 \wedge u_2(t_{k_1}) = 0$  and vice versa, i.e. there will not be any contemporary events.

Using equations (7), (10) and (11) we see that the state equations (2) and (3) may be written as

$$q_1(t_{k+1}) = f_1[\nu_k, \xi_k, q_1(t_k), q_2(t_k)] \quad (12)$$

$$q_2(t_{k+1}) = f_2[\nu_k, \xi_k, q_1(t_k), q_2(t_k)] \quad (13)$$

This formulation again highlights the fact that the system governing the time–variation of probabilities depends on its past state as well as on the random forces  $\nu$  and  $\xi$ .

#### IV. MUTUAL INFERENCE

The constants  $a_1, a_2$  in equations (2) and (3) determine the step–up levels for the event probabilities following the occurrence of a partner event. We now assume that the agents choose  $a_i$  equal to their estimate of the partner probability, i.e.

$$a_1(t_k) = \hat{q}_2(t_k) \quad (14)$$

$$a_2(t_k) = \hat{q}_1(t_k) \quad (15)$$

where  $t_k$  refers to the last event time. Furthermore, let us assume identical base probabilities

$$\bar{q}_1 = \bar{q}_2. \quad (16)$$

We introduce the subsequence  $\{t_l\}_{l \geq 0}$  as the set of time instants at which the Boolean expression in equations (2) and (3) switches. As the conditions for obtaining a one in the Boolean expressions are mutually exclusive this subsequence is well–defined and identical for both agents. This enables us to write

$$q(t_{l+1}) = \begin{bmatrix} 0 & \gamma_2(t_l) \\ \gamma_1(t_l) & 0 \end{bmatrix} q(t_l) \quad (17)$$

using

$$\gamma_i(t_l) = \frac{\hat{q}_i(t_l)}{q_i(t_l)} \quad (18)$$

which adjusts for deviations of the estimates from the true parameters. Notice that the above definition of  $\gamma_i(t_l)$  strictly applies only to phases of “high” probability during which mutual estimation occurs. The adjustment factor will be equal to one during intervals of low probability since we have assumed equal base probabilities. To gain further insight, let us assume that the estimators have converged and thus  $\gamma_i(t_l) = \gamma_i = \text{const.}$  also during the “high” phases. Recall that conversation corresponds to the occurrence of adjacency pairs, i.e. two successive events being produced by the two agents. We refer to a conversation as *perfect* if adjacency pairs appear periodically.

*Proposition:* A sufficient condition for perfect conversation is that the stationary values of the adjustment factors satisfy  $\gamma_1\gamma_2 = 1$ .

*Proof:* If the adjustment factors are constant the system is linear and the stability is governed by the spectral radius  $|z|$  of the matrix in (17). We have that

$$|z| = \sqrt{\gamma_1\gamma_2} = 1 \quad (19)$$

One of the eigenvalues equals  $-1$  which results in a stable 2-periodic solution.  $\square$

Notice that while the dynamics defined in (17) is deterministic over the index  $l$  the actual observed event times (occurring at  $t_l$ ) will, of course, be stochastic and governed by a Bernoulli process.

In general,  $\gamma_i(t_l)$  is a time-varying quantity as it tracks the evolution of the parameter estimates. It is well known that bi-linear systems of the form (17) may become unstable even if the spectral radius does not exceed unity (at every instant of time). Stability has been established in [12] for the case where the true parameters are constant, and –more recently– where they are periodically time-varying [13]. Multiple model based adaptation has been shown to perform well in the case of arbitrary time-variations [14]. The key here is to locate models close to the “true” trajectory (i.e. the values assumed by the time-varying true parameters) so that the speed of convergence is increased [15]. If, on the other hand, the true trajectory is given as a sequence of parameter estimates (generated by the partner agent) we have a *new* situation which we explore presently using qualitative arguments.

We study the convergence path taken by the parameter estimates  $\hat{q}$ . The estimator (4) contains the sum

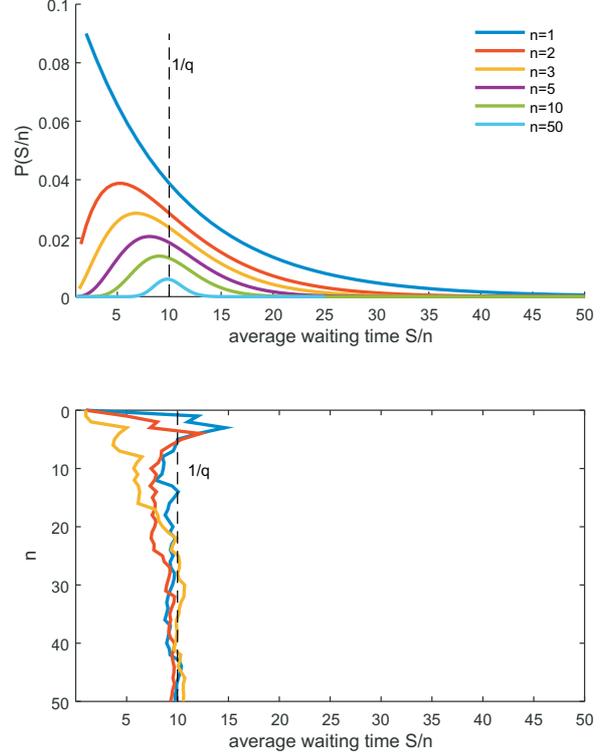


Fig. 2. Above: Distribution of average waiting times for different sample sizes  $n$ . Below: Convergence path of the average waiting times, the true value corresponding to  $1/q$  (as indicated by the dotted bar).

$S_2^* = \sum \bar{t}_{k_2^*} - \bar{t}_{k_1}$  over  $n_2^*$  waiting times. The probability of waiting  $S_2^*$  instants in order to see  $n_2^*$  events is given by

$$P(S = s) = \binom{s-1}{n-1} q^n (1-q)^{s-n} \quad (20)$$

where we have dropped the indexation in order to simplify notation.  $P(S)$  corresponds to the negative binomial distribution and is derived by noting that if the number  $n$  of events is fixed and the last event occurs at instant  $s$  (equivalent to the total waiting time) there are  $\binom{s-1}{n-1}$  possibilities to locate the  $n-1$  other events on the scale from 1 to  $s-1$ . Its mean is given by [16]:

$$\mathbb{E}[S] = \left\lceil \frac{n}{q} \right\rceil \quad (21)$$

The estimator for  $q$  takes the inverse of  $\frac{1}{n}S$  so, by Jensen’s inequality,

$$q = \frac{1}{\frac{1}{n}\mathbb{E}[S]} \leq \mathbb{E}\left[\frac{n}{S}\right] = \mathbb{E}[\hat{q}] \quad (22)$$

In other words,  $\hat{q}$  has a positive bias for any finite number  $n$  of samples (waiting times). The most probable waiting

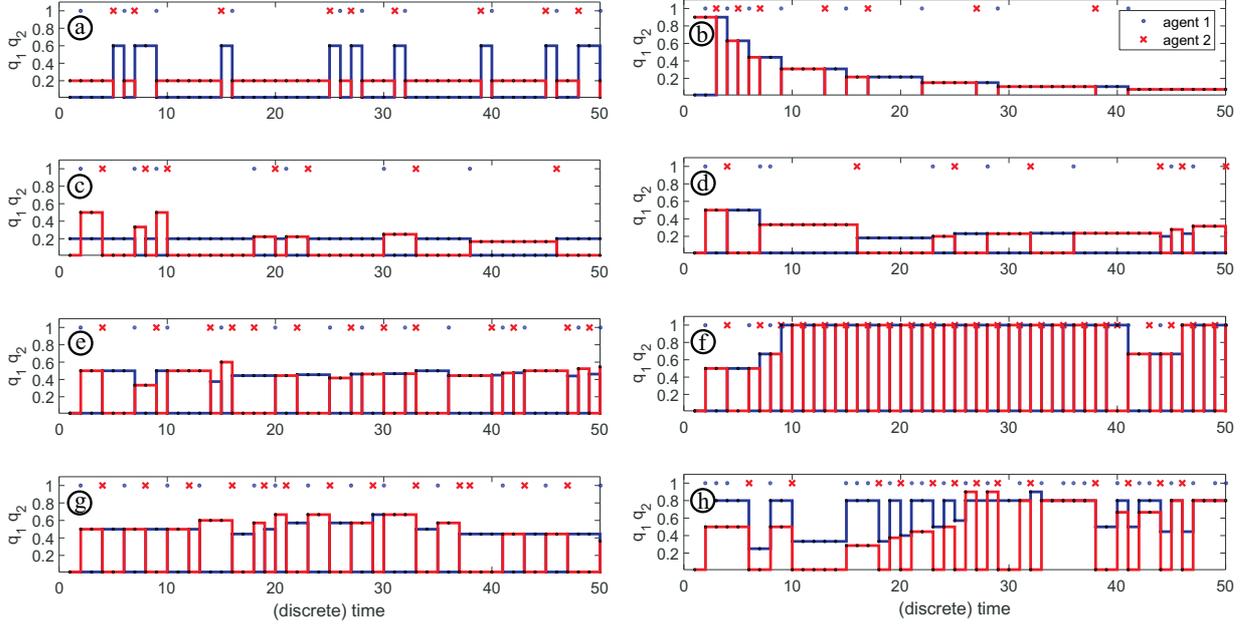


Fig. 3. Model-generated conversations between two agents: Messages are marked as • and ×. The agents use (an estimate of) their partner’s probability in order to determine the intensity of their own message sending. See section IV for a detailed description of the cases (a)-(h).

times can be found in the neighborhood of the (unique) mode of  $P(S)$  given by the largest integer  $s_m$  that satisfies

$$s_m < \frac{n-1+q}{q}. \quad (23)$$

The mode of the distribution of the inverse is simply given by  $1/s_m$  as it refers to the same (most probable) outcome of the corresponding Bernoulli trials. The most probable estimate  $\hat{q}_m$  is then given by

$$\hat{q}_m = \frac{nq}{n-1-q} \geq q \quad (24)$$

where the inequality follows directly from  $0 \leq q \leq 1$ . As the distribution is unimodal, it means that a large amount of probability mass can be found in the neighborhood of  $\hat{q}_m$  which, again, overestimates the true  $q$ . In the limit,  $n \rightarrow \infty$ , we have that  $\hat{q}_m \rightarrow q$  as desired. If agents mutually estimate their event probabilities and employ these estimates in order to generate events we expect an accumulation of biases over successive turn taking cycles. This will eventually result in a very rapid exchange of messages. Note that due to the discrete nature of our problem the average waiting time  $\frac{1}{n}S$  is bounded below by 1 which means that  $\hat{q} \leq 1$ . The propensity to converge to the maximum intensity depends on the

number  $n$  of responses collected by agents in order to perform the inference step. If  $n$  is low the bias in  $\hat{q}$  is large leading to a rapid increase of the “firing” activity. This activity will be occasionally interrupted by periods of low intensity which we expect due to the right tail of the distribution  $P(S)$  of waiting times, see figure (2). For larger values of  $n$  the distribution is less skewed and the bias is lower. We expect steadier values of  $\hat{q}$  and fewer “corrections” (periods of silence) concordant with the notion of a more orderly, less nervous conversations.  $P(S)$  has a positive skew given by

$$\text{Skew}[S] = \frac{2-q}{\sqrt{n(1-q)}} \quad (25)$$

which tends to zero as  $n \rightarrow \infty$ . In fact,  $S$  corresponds to the sum of i.i.d. waiting times (having finite variances) and so, by the central limit theorem,  $P(S)$  converges (in distribution) to the normal distribution. The variance of  $P(S)$  is given by

$$\text{Var}[S] = \frac{n(1-q)}{q^2} \quad (26)$$

which means that  $\text{Var}[\frac{1}{n}S] = \frac{1}{n^2} \text{Var}[S] \rightarrow 0$  as  $n \rightarrow \infty$ . It is not straightforward, however, to translate these properties of  $P(S)$  to  $P(\hat{q})$ .

### Simulation study (figure 3)

The top row in the figure corresponds to the situation where the event probabilities are mutually known. In part (a) the conditions of the proposition are satisfied: the exchange of messages occurs regularly, albeit with different intensities ( $q_1 = 3q_2$ ). In part (b),  $\gamma_1\gamma_2 < 1$  which leads to a gradual fade-out of the conversation. In the second row, we explore the possibility of the two agent probabilities to converge to each other. In part (c) agent 1's probability is fixed and agent 2 gradually adopts this probability as the estimator of  $q_1$  converges. In part (d) both agents adapt their probabilities (towards the partner probability) and we observe phases of mutual agreement. As the overall process is stochastic there still remains the possibility of adjusting the estimates which leads to small deviations from equilibrium (consensus).

In the third row, part (e), the experiment with two adaptive agents is repeated. We observe that the deviations from consensus develop into a small upwards drift of the probabilities. Part (f) contrasts this result with the situation where only the (rolling) last two observations of waiting times are used in the estimator of  $q$ . From the above discussion we know that  $\hat{q}$  will be biased to the upside. In fact, we observe that the agents mutually excite each other due to this bias and we obtain an early lock-in of the conversation at the maximum intensity level. Again, this equilibrium is stable but not attractive as stochastic deviations may lead to a break-down of the exchange (around  $t = 40$  in the simulation).

In the fourth row, the look-back window of both estimators is increased to four. High-intensity phases are also possible but the behavior shown in part (g) is more common and similar to part (e) in which *all* available observations have been used at every instant of time. Finally, in part (h) of the simulation we repeat the experiment but agent 1 maintains a high probability level even after the turn has been given to agent 2 (as if he or she was conducting a simultaneous conversation with a third person). We observe a similar mutual rapprochement during the time windows where agents adhere to the turn-taking schedule but conversation is much more difficult to detect.

### V. CONCLUDING REMARK

Our model imposes a strict turn-taking schedule to the exchange of messages between agents. Despite this constraint a rich set of temporal patterns is generated due to the mutual adaptation of intensities by which

messages are being sent. The model complements efforts to represent the content of conversations sparsely through the introduction of latent aspects or topics, see [17], [18]. In a related project, we employ these techniques in conjunction with the model presented in this paper in order to predict the evolution of topic intensities.

### VI. ACKNOWLEDGEMENT

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